Implications of Temperature on the Efflux of Milk Through Varying Pipe Specifications

C. Barker Carlock, Sean McSherry, and Ben Tschan

Department of Chemical and Biomolecular Engineering, Lafayette College

Introduction

The pasteurization of milk—a Non-Newtonian suspended colloid—occurs at high temperatures. Understanding the rheological properties of milk is therefore essential for the commercial production of dairy products. To minimize the rate at which milk is bottled in post-pasteurization, the dimensions of piping, as well as temperature and its effects on viscosity must be analyzed.

Abstract

Using Design of Experiment Analysis, experimental results were used to delineate the factors that affect overall efflux time of a draining tank. Incorporating the significant factors yielded results that display the quickest and most effective efflux times, thus allowing for it to be determined when it is most appropriate to cool down the milk. To achieve the shortest efflux time of 34.9sec, it should be bottled with a pipe of diameter 0.001565 m, length 0.0305 m, and cooled post-packaging.

Theory

Power Law Fluid

A power law fluid has a viscosity that changes as a function of shear rate. This relationship is described by Equation 1.

$$(1)\quad \mu_{app} = K\left(\frac{dv}{dr}\right)^{n-1}$$

μ_{app} = Apparent viscosity (Pa·s)
K = Flow consistency index (Pa·s^n)
n = Flow behavior index
dv/dr = Shear rate (s^{-1})

If n>1, the viscosity of the fluid ↑ as shear rate ↑, and is subsequently labeled shear thickening. If n<1, the viscosity of the fluid ↓ as shear rate ↑, and thus labeled shear thinning. If n=1, the fluid is Newtonian.

Pipe Shear Rate

To calculate shear rate within various pipe diameters Equation 2 may used. This shear rate can be related to viscosity with Equation 1.

$$(2)\quad \left(\frac{dv}{dr}\right) = \frac{8v}{D}$$

v = Velocity (m/s)
D = Diameter of pipe (m)
dv/dr = Shear rate (s^{-1})

Efflux Time in a Draining Tank

The amount of time required to drain vessels is known as efflux time. Using the Hagen-Poiseuille Law, Equation 3 may be derived to calculate theoretical efflux time through Figure 2.

$$(3)\quad \ln\left[\frac{H_f+L}{H_i+L}\right] = \frac{-gR_0^4}{8\eta L R^2}t_R$$

H = Height of liquid (f=final, i=initial) (m)
L = Length of pipe (m),
R = Radius (R_0= of pipe, R= of tank) (m)
t_R = Efflux time of pipe (s)

Acknowledgements

This project would not have been possible without the support of Dr. Polly Piergiovanni, Tom DeFazio, and the Department of Chemical and Biomolecular Engineering.

Contact Information

C. Barker Carlock	Sean McSherry	Ben Tschan
carlockc@lafayette.edu	mcsherrs@Lafayette.edu	tschanb@lafayette.edu
(214) 770-5772	(267) 337-3033	(914) 960-5959

Experimental Methods

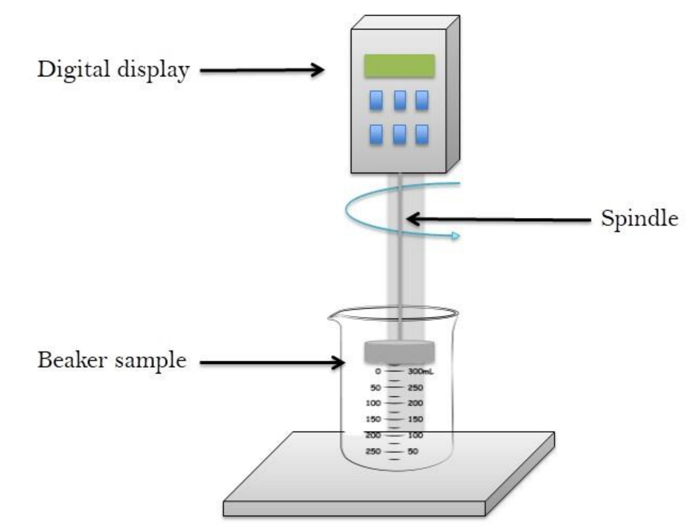


Figure 1: Brookfield Viscometer

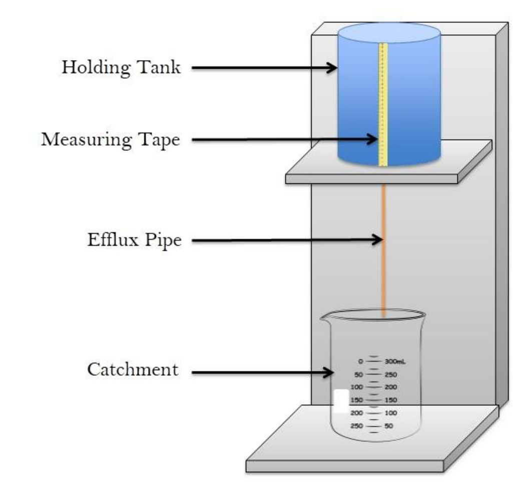


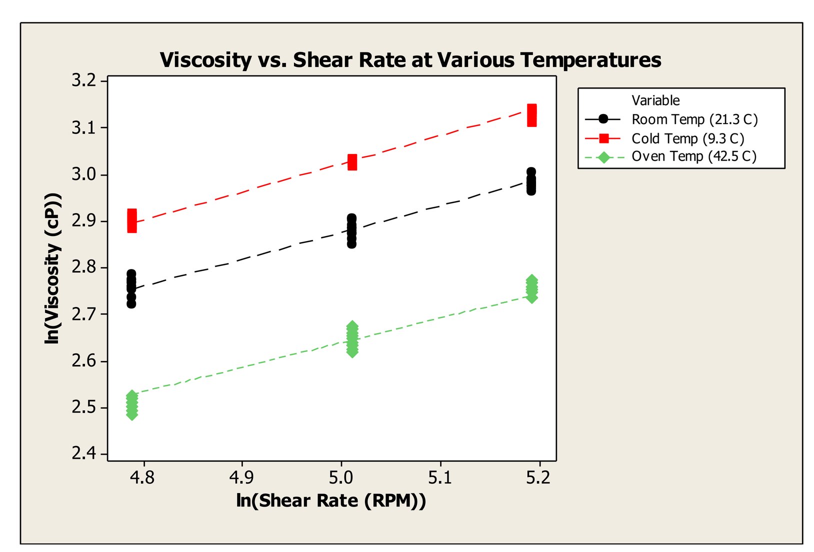
Figure 2: Efflux Tank Configuration

Whole milk was placed at 9.3°C, 21.3°C, 42.5°C. → Viscosities where obtained using a Brookfield Viscometer

At each temperature, 12 trials were recorded at 3 different shear rates → 12 trials for each temperature were put through the efflux apparatus

Efflux times were obtained using a stopwatch ← 3 trials for each temperature with varying pipe length and diameter (short : long, small : large)

Results



Temperature	Linear Equation
Refrigerator	$\ln(\mu_{app}) = 0.2130 + 0.5622\ln\left(\frac{dv}{dr}\right)$
Room	$\ln(\mu_{app}) = 0.1369 + 0.5478\ln\left(\frac{dv}{dr}\right)$
Oven	$\ln(\mu_{app}) = -0.3624 + 0.6002\ln\left(\frac{dv}{dr}\right)$

Refrigerator Trials (9.3 C)

Pipe Dimensions	Theoretical Efflux Time	Avg. Exp. Efflux Time	% CV	Avg. % Error
Short Length/Small Diameter	98.0	127	1.07%	29.1%
Short Length/Large Diameter	29.1	47.0	1.04%	62.3%
Long Length/Small Diameter	157	144	1.54%	9.15%
Long Length/Large Diameter	49.4	48.0	1.38%	3.05%

Room Trials (21.3 C)

Pipe Dimensions	Theoretical Efflux Time	Avg. Exp. Efflux Time	% CV	Avg. % Error
Short Length/Small Diameter	98.0	99.4	1.60%	1.81%
Short Length/Large Diameter	27.8	39.9	0.752%	43.9%
Long Length/Small Diameter	163	106	1.45%	34.9%
Long Length/Large Diameter	49.4	37.8	1.35%	23.6%

Oven Trials (42.5 C)

Pipe Dimensions	Theoretical Efflux Time	Avg. Exp. Efflux Time	% CV	Avg. % Error
Short Length/Small Diameter	85.6	77.6	2.33%	9.30%
Short Length/Large Diameter	23.2	34.9	3.77%	54.8%
Long Length/Small Diameter	149	76.8	2.77%	48.4%
Long Length/Large Diameter	37.3	38.1	1.71%	2.83%

Analysis

- Viscosity of milk increased as shear rate was increased.
  - ǂ Understanding that high fat content milk is a suspended colloid of triglycerides, deflocculation is hypothesized as the justification for shear-thickening properties.
- DOE analysis proved that temperature, pipe length, and pipe diameter all affect efflux time
- Milk at the oven temperature flowing through the shortest pipe and largest diameter proved to have the fastest efflux times
  - ǂ As T ↑, μ↓

Sources of Error and Improvements

- Due to the size of the beaker and spindle used, the Brookfield viscometer could have experienced end effects purporting the shear-thickening properties observed (see Figure 3).
- Due to the apparatus itself, expansion and contraction take place at the inlet and outlet of the pipe, affecting the overall flow assumptions (see Figure 4).
- Within Equation 3, the pressures were considered to be equal, although in reality, a pressure drop existed
- Human error was introduced through the practitioner's manipulation of a stopwatch as well as the measurement of column height due to the presence of milk foam.

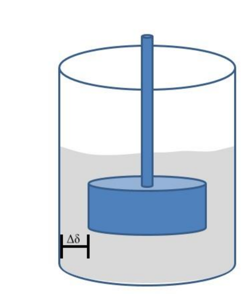


Figure 3: Viscometric End Effects

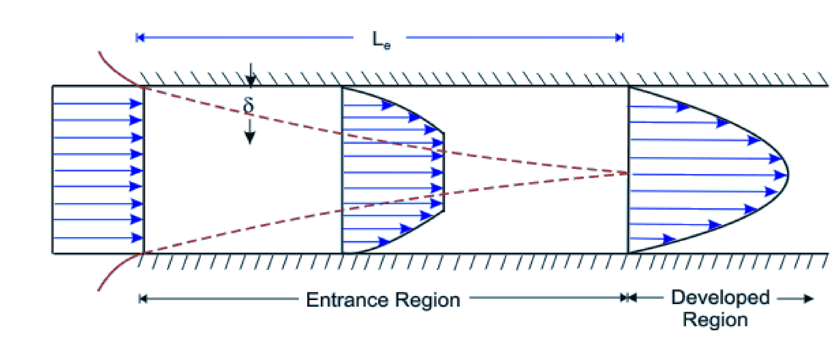


Figure 4: Entrance Diagram

Conclusion and Future Work

- In conclusion, the efflux times of milk decreased as the temperature increased. Furthermore, the combination of short pipe length and large diameter resulted in the fastest efflux times in each of the varying temperature trials.
- In order to increase the rate at which milk is processed post-pasteurization, it should be bottled with a short pipe containing a large diameter, and cooled after packaging.
- Future work should be conducted with the variation of fat content within milk to better understand its effects on viscosity.

References

Bird, R. Byron, Edwin N. Lightfoot, and Daniel J. Klingenberg. *Introductory Transport Phenomena*. 1st Edition. Wiley. 2014.
"Milk Composition and Synthesis Resource Library" http://ansci.illinois.edu/static/ansc438/Milkcompsynth/milkcompsynthresources.html
Density of Milk. *The Physics Factbook.* http://hypertextbook.com/facts/2002/AliciaNoelleJones.shtml
Trinh, Khanh. *The Instantaneous Wall Viscosity in Pipe Flow of Power Law Fluids: Case Study for a Theory of Turbulence in Time-Independent Non-Newtonian Fluids.* Institute of Nutrition and Human Health.
"Entry Flow in a Duct" http://nptel.ac.in/courses/112104118/lecture-31/31-1_entry_flow.htm